DEMONSTRATION *of* NEWTON's THEOREM *for the* CORRECTION *of* SPHERICAL ERRORS *in the* OBJECT GLASSES *of* TELESCOPES. *By the Rev.* MATTHEW YOUNG, *D.D. S.F.T.C.D. and M.R.I.A.*

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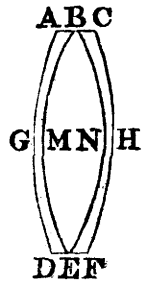
SIR Iſaac Newton, in his Optics, B. I. Part I. p. 68, had obſerved, that were it not for the different refrangibility of the rays of light, teleſcopes might be brought to a great degree of perfection, by compoſing the object glaſs of two glaſſes with water between them, in the following manner:



LET *ADFC* repreſent the object glaſs compoſed of two glaſſes *ABED*, and *BEFC* alike convex on the outſides *AGD* and *CHF*, and alike concave on the inſides *BME*, *BNE*, with water

in the concavity *B M E N*. Let the ſine of incidence to the ſine of refraction of glaſs into air be as *I* to *R*, and out of water into air as *K* to *R*, and by conſequence out of glaſs into water as *I* to *K*: and let the diameter of the ſphere to which the convex ſides *A G D* and *C H D* are ground be *D*; and the diameter of the ſphere to which the concave ſides are ground be to *D*, as the cube root of *K K—K I* to the cube root of *R K—R I*; by which means the refractions on the concave ſides of the glaſſes will *very much* correct the errors of the refractions on the convex ſides, ſo far as they ariſe from the ſphericalneſs of the figure.

M. Euler, improving on this hint, was not without hopes of being able, by the ſame artifice, to prevent the diſperſion which is occaſioned by the difference of refrangibility; and publiſhed a memoir on that ſubject in the Berlin Tranſactions for the year 1747. This memoir excited the attention of Mr. Dollond, and gave riſe to that controverſy, which terminated ſo happily in the glorious diſcovery of the Achromatic Teleſcope.

It is ſingular that this conſtruction of the firſt compound object glaſs, though ſo principal a ſubject of enquiry, ſhould never have been demonſtrated in the progreſs of this controverſy. Count Redern, in his memoir on "The influence which the famous "Newton attributes to the different refrangibility of the rays of "light on refracting Teleſcopes," in the Berlin Tranſactions for the year 1760, obſerves, that "it is to be lamented, that "Newton has not given us the demonſtration of this admi-"rable conſtruction, the diſcovery of which would not be in-

"ferior

" ferior to that of the different refrangibility of light; and " that he has not informed us whether he had tried it."

In the quarto and octavo editions of the Optics, in Doctor Clarke's Latin tranſlation, in Doctor Horſeley's edition, in Martin's Phil. Brit. vol. III. p. 62. and in the memoir by Count Redern above referred to, the ratio of the radii is determined in the ſame manner as deſcribed above. And yet there is no doubt but that ſome error muſt have crept into the text; for the quantities $KK—KI$ and $RK—RI$ are to each other as K to R, and therefore the ratio of the radii, according to this expreſſion, depends ſolely on the refraction between water and air, without involving any conſideration of the refractive power of the ambient medium that incloſes the water, which is evidently falſe. Neither can it be ſuppoſed that Newton would have given the ratio in this complex form, which is intuitively reducible to an expreſſion ſo much more ſimple. Mr. Harris in his Optics, and Doctor Prieſtley in his Hiſtory of Viſion, &c. deſcribe the contrivance in general, as conſiſting in cementing together two like concavo-convex glaſſes, with water between them, the radii of whoſe ſurfaces ſhall have a certain ratio to each other; but neither of them tell us what that ratio ſhould be. An inveſtigation of the demonſtration of this conſtruction is therefore deſirable, not only on its own account, but alſo as it may lead to a correction of Newton's text, which, from what has been obſerved, appears manifeſtly to be corrupt.

If the ratio of I to R denote the ratio of the ſine of incidence to the ſine of refraction out of glaſs into air, D the radius of the ſpherical

ſpherical ſurface, and y the ſemi-aperture of the lens, Newton has demonſtrated, that when parallel rays fall on the plane ſide of a plano-convex lens, the lateral aberration at the focus, ariſing from the ſphericalneſs of the figure, will be equal to $\frac{R^2 y^3}{2I^2 D^2}$; which divided by $\frac{R D}{R-I}$, the diſtance of the focus from the centre of the lens, gives $\frac{\overline{R^2-RI}.y^3}{2I^2 D^3}$ for the angle which the lateral aberration at the focus ſubtends at the centre of the ſurface. Now when the ray paſſes from the glaſs into the included water, and from the water into the glaſs, it is evident that the refractions, and conſequently the ſpherical aberrations, are contrary to thoſe at the firſt and laſt ſurface; and therefore, that all aberration ſhould vaniſh, the lateral aberrations from glaſs into air and from glaſs into water (as being the principal, and *nearly* equal to thoſe from air into glaſs, and from water into glaſs) ſhould ſubtend equal angles at the common centre of the ſpherical ſurfaces. The ſine of incidence out of water into air is to the ſine of refraction as K to R, and by conſequence out of glaſs into water as I to K; therefore if we ſuppoſe the rays, in paſſing from the glaſs into the water, to fall parallel on the water, the angle which the lateral aberration of the rays at the focus ſubtends at the centre of the refracting ſurface will be equal to $\frac{\overline{K^2-KI}\times y^3}{2I^2 d^3}$; which being made equal to $\frac{\overline{R^2-RI}\times y^3}{2I^2 D^3}$, we have d^3 to D^3 as $KK-KI$ to $RR-RI$. Though the rays do not fall parallel on the water, yet the error cauſed by their divergence

divergence or convergence will produce no difference in the concluſion of any conſequence: See Emerſon's Dioptrics, Cor. 7 & 8. Prop. 40. B. 3. And that perfect accuracy was not affected by Newton is evident from his words, " will very much correct the " errors of refraction, ſo far as they ariſe from the ſphericalneſs " of the figure."

We may obſerve, from the expreſſion laſt deduced, that the ratio of the radii of the ſurfaces depends as well upon the refraction between air and glaſs, as upon the refraction between glaſs and water. We may alſo obſerve, that in the laſt term of the analogy, as it ſtands in Newton's text, we ſhould read $R\,R$ inſtead of $R\,K$; which renders the whole conſiſtent, and removes a manifeſt corruption.

If the ſurfaces be not concentrical, each glaſs becoming a meniſcus, no error will thence ariſe; becauſe the centre of the concave ſurface will be as much farther from the focus of the compound lens on one ſide, as it is nearer to it on the other; and therefore the correction by one ſurface will be as much too great, as too little by the other, ſo that the ſum of both aberrations will, in this caſe likewiſe, very nearly vaniſh.